

One and two dimensional Bose-Einstein condensation of atoms in dark magneto-optical lattices

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The motion of atoms in a dark magneto-optical lattice is considered. This lattice is formed by a non-uniformly polarized laser field in the presence of a static magnetic field. Cold atoms are localized in the vicinity of points where dark states are not destroyed by a magnetic field. As a result the optical interaction tends to zero (dark or gray lattices). Depending on the field configuration such a lattice can exhibit one or two-dimensional periodicity. It is shown that in 1D and 2D dark magneto-optical lattices the effects connected with the Bose-statistics of particles can be observed under the temperature $10^{-5} - 10^{-6} K$ and densities $10^{11} - 10^{12} cm^{-3}$, those are usual for current experiments on laser cooling.

I. INTRODUCTION

After the outstanding experiments on Bose-Einstein condensation (BEC) of an atomic gas in a magnetic trap [1–3] studies of phenomenon originated from quantum statistics of particles became one of the main subjects in contemporary atomic physics. Achieving of BEC by optical means is of especial interest [4–7]. On the one hand this allows deeper understanding of the nature and physical properties of condensates under various conditions. On the other hand, the works in this direction may lead to new effective methods of super-deep cooling. It is worth noting, that in the refs. [1–3] laser fields play an auxiliary role and they are used for precooling down to sufficiently low temperatures with successive evaporating cooling in a magnetic trap, when in the final stage of experiments optical fields are absent and the main part of atoms escape from a trap under evaporative cooling. One of the main components of the optical methods [4,6] is the formation of a non-dissipative optical or magneto-optical potential, when atoms scatter photons with very low rates. There exist two principal ways to solving this problem. The first one consists in the use of far-off-resonance light fields with the high intensity [4,6,8]. In this case the potential depth of order of $10^3 \varepsilon_r$ ($\varepsilon_r = (\hbar k)^2 / 2M$ is the single-photon recoil energy) and the rate of spontaneous photon scattering about $1 s^{-1}$

are achieved [6,7]. Another way is connected with the use of coherent population trapping (CPT) phenomena in near-resonance light fields. As is known [9], under the resonance interaction of a polarized radiation with atoms, having optical transitions $F_g = F \rightarrow F_e = F$ with F an integer and $F_g = F \rightarrow F_e = F - 1$ (g and e denote the ground and excited states respectively), there exist dark states. These states are coherent superposition of the ground-level Zeeman substates, which is fully decoupled with light $(\hat{\mathbf{d}} \cdot \mathbf{E})|\psi_{nc}\rangle = 0$. Due to optical pumping atoms are trapped in these states and do not scatter light. The use of fields with a polarization gradient allows to create a potential in dark states (dark potential). Although for dark states ac Stark shift vanishes, dark potentials can be created by the atomic translational motion (so-called gauge or geometric potentials) [10–12], or by applying of a static (magnetic or electric) field. In the later case the atomic multipole moments are spatially non-uniform, that leads to the coordinate dependence of the interaction energy with a static field. As a result, a periodic potential is formed. From the other hand, static fields induce precession of multipole moments and destroy dark states. However, this effect can be suppressed to negligible values due up to two factors:

1. We can choose a specific geometry of fields [13] where atoms are localized near the points, where dark states are not destroyed by a static field.
2. The use of high-intensity laser field allows to lack atoms in dark states. In the result, as is shown in ref. [14], the rate of spontaneous scattering is inverse proportional to the light intensity.

A quantitative treatment of dark magneto-optical lattices in the non-dissipative regime at high-intensity laser field had been developed in ref. [14] in the one-dimensional case. It had been shown that the potential depth is determined by the ground-state Zeeman splitting $\hbar\Omega$, while the period of lattice is of order of the light wavelength λ . Thus, we can obtain a very deep potential with a large spatial gradient. Both these reasons lead to the large energy separation between vibrational levels $\sim \sqrt{\hbar\Omega\varepsilon_r}$, which can exceed the laser cooling temperature. Under these conditions, atoms being in the lower vibrational levels can be localized within a very small distance $\sim \lambda\sqrt{\varepsilon_r/\hbar\Omega}$. We stress that tunneling between wells is

exponentially small (by factor $\exp(-\sqrt{\hbar\Omega/\varepsilon_r})$), that allows to consider atoms in each of wells as independent systems.

In the present paper, with $F_g = 1 \rightarrow F_e = 1$ transition as an example, one- and two-dimensional non-dissipative magneto-optical lattices are considered with especial attention to the formation of atomic structures with lower dimensions ($2D$ – planes, $1D$ – lines). In such a lattice the spontaneous scattering of photons is strongly reduced and the main dissipative mechanism is elastic interatomic collisions. In the framework of an ideal Bose-gas model it is shown that under applying of an additional weak confining potential, the condensation is possible at the temperatures and densities typical for the current experiments on laser cooling.

It is worth noting, in the low-saturation limit, that corresponds to the dissipative regime, dark magneto-optical lattices had been theoretically [15] and experimentally [16] studied. Sub-Doppler cooling down to $20 \varepsilon_r$ had been predicted and observed. Experimental evidence of the translational motion quantization had been presented.

II. NON-DISSIPATIVE DARK MAGNETO-OPTICAL LATTICE

Let us consider a gas of Bose-atoms with total angular momenta $F_g = 1 \rightarrow F_e = 1$ in a resonant spatially nonuniform monochromatic laser field

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) e^{-i\omega t} + c.c. \quad (1)$$

in the presence of a static magnetic field \mathbf{B} . As is known [6], for all $F \rightarrow F$ (F is an integer) transitions there exist dark CPT-states uncoupled with a laser field (1):

$$(\hat{\mathbf{d}} \cdot \mathbf{E}(\mathbf{r})) |\psi_{nc}(\mathbf{r})\rangle = 0, \quad (2)$$

where $\hat{\mathbf{d}}$ is the dipole moment operator. In the case under consideration $F = 1$ this state has the form [17]:

$$|\psi_{nc}(\mathbf{r})\rangle = \frac{1}{|\mathbf{E}(\mathbf{r})|} \sum_{q=0,\pm 1} E^q(\mathbf{r}) |g, \mu = q\rangle, \quad (3)$$

where $E^q(\mathbf{r})$ are the field components in the spherical basis $\{\mathbf{e}_0 = \mathbf{e}_z, \mathbf{e}_{\pm 1} = \mp(\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}\}$. The state (3) is a superposition of the ground-state Zeeman wave functions $|g, \mu\rangle$. We note that in the general case this state is neither eigenvector of the Hamiltonian of the interaction with the magnetic field $\hat{H}_B = -(\hat{\mu} \cdot \mathbf{B})$ nor of the kinetic energy Hamiltonian $\hat{H}_K = \hat{p}^2/2M$. Thus, the state $|\psi_{nc}(\mathbf{r})\rangle$ is not strictly stationary. However, the corrections to the wave function resulting from the translational motion and the magnetic field can be considered as small perturbations with respect to the atom-light interaction under the conditions:

$$V(\mathbf{r})\sqrt{G} \gg k\bar{v}, \Omega, \quad (4)$$

where $V(\mathbf{r}) = |\langle \hat{d} \rangle \mathbf{E}(\mathbf{r})|/\hbar$ is the Rabi frequency, $G = V^2(\mathbf{r})/[\gamma^2/4 + \delta^2 + V^2(\mathbf{r})]$ is the effective saturation parameter, δ is the detuning, γ is the inverse lifetime of the excited state and \bar{v} is the average atomic velocity. In this case the main part of atoms is pumped into the dark state $|\psi_{nc}(\mathbf{r})\rangle$. Under the conditions (4) the relative populations in the CPT-state n_{nc} and in the excited state n_e obey the relation:

$$(1 - n_{nc}) \sim n_e \sim \left(\frac{\max\{k\bar{v}, \Omega\}}{V(\mathbf{r})\sqrt{G}} \right)^2 \ll 1.$$

Then the evolution of a single atom can be described, with the same accuracy, by the effective Hamiltonian:

$$\hat{H}_{eff}^{(1)} = \langle \psi_{nc}(\mathbf{r}) | (\hat{H}_K + \hat{H}_B) | \psi_{nc}(\mathbf{r}) \rangle. \quad (5)$$

Using the explicit form of the CPT-state (3), we write the one-particle Hamiltonian (5) as a sum of four terms:

$$\hat{H}_{eff}^{(1)} = \frac{\hat{p}^2}{2M} + U(\mathbf{r}) + \frac{1}{2M} \{(\mathbf{A}(\mathbf{r}) \cdot \hat{\mathbf{p}}) + (\hat{\mathbf{p}} \cdot \mathbf{A}(\mathbf{r}))\} + W(\mathbf{r}). \quad (6)$$

The first term is the kinetic Hamiltonian. The second one is the magneto-optical potential:

$$U(\mathbf{r}) = \hbar\Omega \frac{i(\mathbf{B} \cdot [\mathbf{E}(\mathbf{r}) \times \mathbf{E}^*(\mathbf{r})])}{|\mathbf{B}| |\mathbf{E}(\mathbf{r})|^2}, \quad (7)$$

which is independent of the amplitude and phase of light field. The last two corrections in Eq.(6) caused by the translational motion of atom. The first of these is of the order of $k\bar{v}$ and can be interpreted as the interaction with the effective vector-potential:

$$A_j(\mathbf{r}) = -i\hbar \left(\frac{\mathbf{E}^*}{|\mathbf{E}|} \cdot \frac{\partial}{\partial x_j} \frac{\mathbf{E}}{|\mathbf{E}|} \right). \quad (8)$$

The second correction is of the order of the recoil energy $\hbar\omega_r$ and makes a contribution into the atomic potential energy:

$$W(\mathbf{r}) = \frac{\hbar^2}{2M} \sum_j \left| \frac{\partial}{\partial x_j} \frac{\mathbf{E}}{|\mathbf{E}|} \right|^2. \quad (9)$$

If the Zeeman splitting obeys the conditions

$$\Omega \gg k\bar{v}, \varepsilon_r/\hbar, \quad (10)$$

then the last two terms in Eq.(6) are negligible, i.e.

$$\hat{H}_{eff}^{(1)} \approx \frac{\hat{p}^2}{2M} + U(\mathbf{r}).$$

In this case the problem is reduced to the motion of a particle in the magneto-optical potential (7) only. The depth of this potential is determined by the ground-state

Zeeman splitting $\hbar\Omega$ (below we suppose $\Omega > 0$) and its period is of order of the light wavelength λ .

As is well known, in a periodic potential the energy spectra has the band structure. However, due to the condition (10) the tunneling is negligible for the lower bands, i.e. the strong binding of the particle in a single well is realized. It can be shown that the widths of the lower bands are exponentially small by the factor $\exp(-\sqrt{\hbar\Omega/\varepsilon_r})$ with respect to the energy separation between bands. The positions of these bands are determined (with good accuracy) by a harmonic expansion of the potential in the vicinity of the well bottom:

$$U(\mathbf{r}) \approx \hbar\Omega k^2 \sum_{i,j=1,2,3} C_{ij} x_i x_j. \quad (11)$$

As is seen, the separation between lower levels is of the order of $\sqrt{\hbar\Omega\varepsilon_r}$ at the eigenvalues of \hat{C} of order of 1.

As it has been shown in ref. [14], atoms being in the lower vibrational levels scatter photons with extremely low rates:

$$\tau^{-1} \sim \gamma \left(\frac{\Omega}{V} \right)^2 \sqrt{\frac{\varepsilon_r}{\hbar\Omega}} \ll \gamma.$$

Here the factor $(\Omega/V)^2 \ll 1$ is directly connected with CPT-effect, when in a strong light field the probability of leaving of a dark state is inverse proportional to the light intensity. The additional multiplier $\sqrt{\varepsilon_r/\hbar\Omega} \ll 1$ arises from the localization of atoms in the vicinity of points, where dark state are not destroyed by a magnetic field.

III. IDEAL BOSE-GAS IN DARK MAGNETO-OPTICAL LATTICES

As it has been indicated in the Introduction, the spontaneous photon scattering in dark magneto-optical lattices can be strongly suppressed. Hence the main dissipative mechanism, leading to the thermal equilibrium, is the elastic interatomic collisions. This allows to apply the methods of statistical physics to study of a stationary state of atoms, which corresponds to the thermodynamic equilibrium. In the present paper we restrict our treatment by the ideal gas model when the contribution of atom-atom interactions into the system energy is negligible. At the same time collisions are implicitly taken into account as a reason of the thermal equilibrium.

BEC is one of the most interesting phenomenon arising in a Bose gas under sufficiently low temperatures. As it has been shown by recent studies, the character and parameters of the phase transition essentially depend on the system dimensions and on the presence of an external confining potential. So, it is well-known [18], that in the case of a free gas BEC in the 1D and 2D cases is absent. However, as it has been recently shown in refs. [19,20] in both 1D and 2D cases BEC becomes possible under applying a confining potential. Moreover, the conditions

for the BEC achievement are appreciably less stringent than in the 3D case.

The non-dissipative dark magneto-optical lattice considered in the previous section are promising tools for studies of BEC in systems with lower dimensions. Let us consider the concrete examples.

A. 1D lattice – 2D condensation

The simplest realization of 1D dark magneto-optical lattice is the $\text{lin} \perp \text{lin}$ light field configuration plus a magnetic field directed along the wave propagation direction (see in fig.2). Here the dark magneto-optical potential has the form [14]:

$$U = -\hbar\Omega \cos(2kz). \quad (12)$$

Atoms are localized in the planes $z_n = \lambda n/2$, where the field has the σ_- polarization and the dark state coincides with the Zeeman substate $|F_g = 1, \mu = -1\rangle$. The lower energy levels of the potential (12) correspond to the localization of atoms in the vicinity of the single well bottom, when tunneling between wells is negligible. Basing on a harmonic approximation, one can find the energy separation between the lower levels $\Delta\varepsilon = \sqrt{8\hbar\Omega\varepsilon_r}$. Then under the temperatures

$$k_B T < \sqrt{8\hbar\Omega\varepsilon_r}$$

atoms are in the vibrational ground state. In the other words, the translational motion of atoms along z is frozen. For instance, for the D1-line of ^{87}Rb ($\lambda = 787\text{nm}$) under the magnetic field amplitude $B \sim 4\text{G}$ freezing is achieved at $T < 10^{-5}\text{K}$. Due to the absence of tunneling, each of localization planes can be considered as an independent thermodynamic and mechanical system, where particles freely move along the x and y axes.

If an additional confining (in the xy -plane) potential is applied (for example, far-off-resonance optical shift), then BEC can be reached in a single plane. As it was shown in ref. [20], for 2D harmonic potential with the frequency f the expression for the transition temperature is given by:

$$N = 1.6 \left(\frac{k_B T_c}{\hbar f} \right)^2, \quad (13)$$

where N is the number of atoms in a single plane. For an atomic sample with the density $n \sim 10^{11} - 10^{12}\text{cm}^{-3}$ and with the size $L \sim 10^{-1}\text{cm}$ we have $N \sim 10^5 - 10^6$ for each plane at the periodicity about 10^{-4}cm . Then for a confining potential with $f \sim 10^3\text{Hz}$ the transition temperature $T_c \sim 10^{-5}\text{K}$ is in few orders higher than the transition temperature in 3D magnetic traps [1–3].

In the case under consideration BEC can be observed, for instance, by the time-of-flight measurements of the

atomic momentum distribution after turning off of a confining potential. It should be noted that the phases of condensates in each planes are independent. So, if the lattice is considered as whole, we have the quasicondensation only.

B. 2D lattice – 1D condensation

The example of three-beam field configuration where the 2D dark magneto-optical potential is formed is shown in fig.3. Here the three wave-vectors lie in the xy -plane and make an angle $2\pi/3$ one with another. The linear polarizations of beams make the same angle $\phi \neq 0, \pi/2$ with the z -axis. This angle ϕ can be varied in a wide domain. The main reason for the inequality $\phi \neq \pi/2$ is the fact that in the case of $\phi = \pi/2$ there exist lines, where the field amplitude is equal to zero due to the interference. In the vicinity of these lines the CPT conditions (4) are violated. The case of $\phi = 0$ is not suitable due to the absence of magneto-optical potential (7).

Atoms are localized in the vicinity of straight lines, where the field has σ_- circular polarization. In the same manner as in the previous section, one can show that under sufficiently low temperatures $k_B T < \sqrt{\hbar \Omega \varepsilon_r}$ the translational degrees of freedom along x and y are frozen out. Each of localization lines can be considered as an independent 1D system. Under applying of a confining (along z) harmonic potential we have the condensation at the temperature [20]:

$$N = \frac{k_B T_c}{\hbar f} \log \left(\frac{2k_B T_c}{\hbar f} \right), \quad (14)$$

where N is the number of atoms in a line, f is the oscillation frequency.

For an atomic sample with the density $n \sim 10^{12} \text{cm}^{-3}$ and the size $L \sim 10^{-1} \text{cm}$ we have $N \sim 250$ for each line at the periodicity about 10^{-4}cm . Then under $f \sim 10^3 \text{Hz}$ we find $T_c \sim 10^{-6} \text{K}$, that is typical for laser cooling experiments.

Note that like the case of 1D lattice, here the quasicondensation is possible only, if the whole gas volume is considered.

IV. CONCLUSION

We have considered dark magneto-optical lattices in the regime, when the Rabi frequency of laser field is much greater than the Zeeman splitting of the ground state. In such a regime the lattice is essentially non-dissipative. We have shown that 2D and 1D atomic structures can be formed in these lattices. Then we have studied the Bose-Einstein condensation in the lower-dimensional systems in the framework of the ideal gas model. It was predicted that BEC is possible in both 2D and 1D cases under the temperatures $10^{-6} - 10^{-5} \text{K}$

and densities $10^{11} - 10^{12} \text{cm}^{-3}$. Concluding we note that the above developed approach can be applied to any non-dissipative optical lattice with the sufficient large depth. The example is a far-off-resonance optical lattice [5,8].

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FIG. 1. Scheme of lower-energy vibrational structure in a dark magneto-optical lattice.

FIG. 2. The $lin \perp lin$ light field configuration, when one-dimensional dark magneto-optical potential is formed. Atoms freely move in the planes (parallel to the xy -plane), where the field has the σ_- polarization.

FIG. 3. The three-beam light field configuration, when two-dimensional dark magneto-optical potential is formed. Atoms freely move in the lines (parallel to the z -axis), where the field has the σ_- polarization.

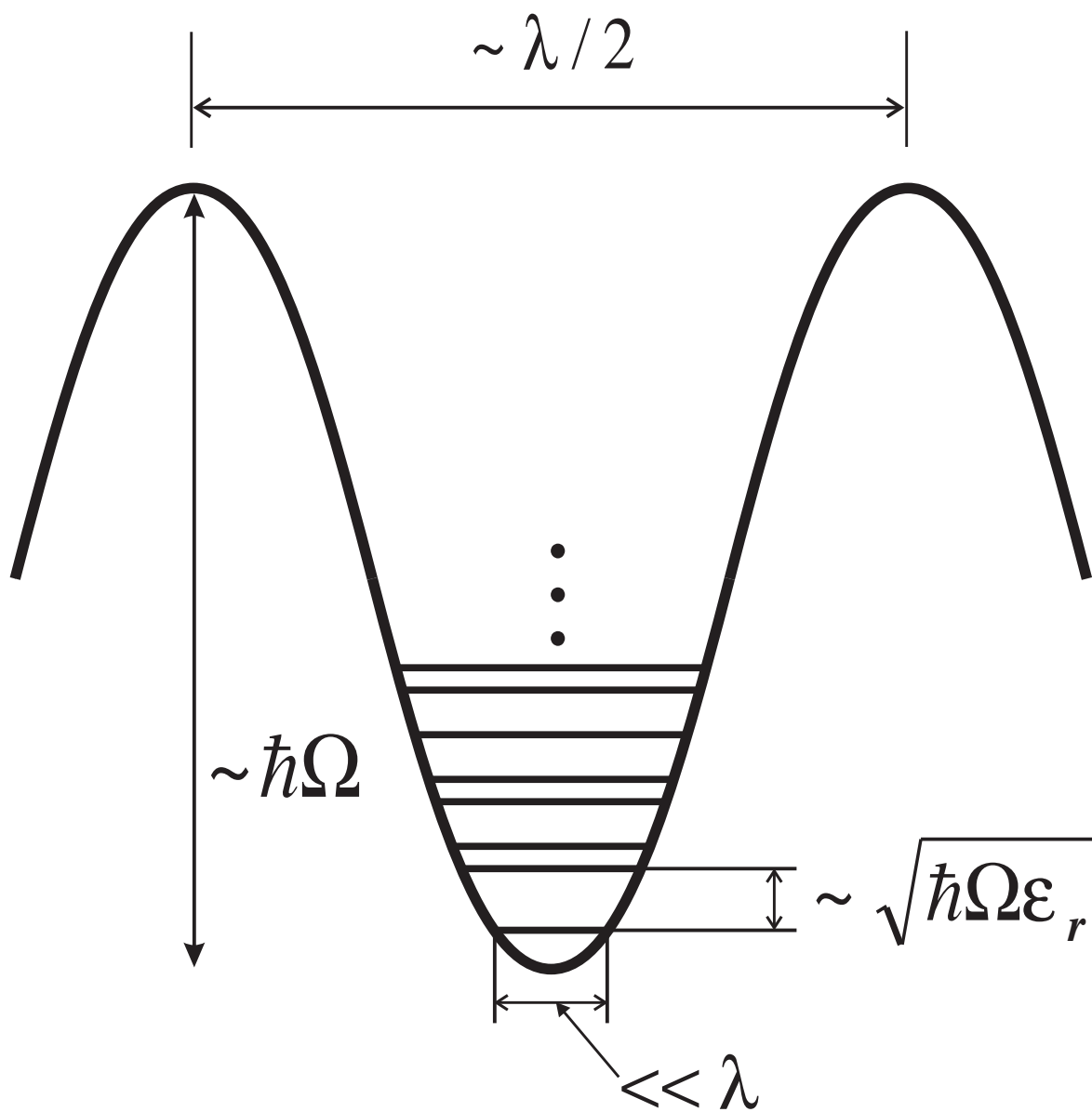


Fig.1.

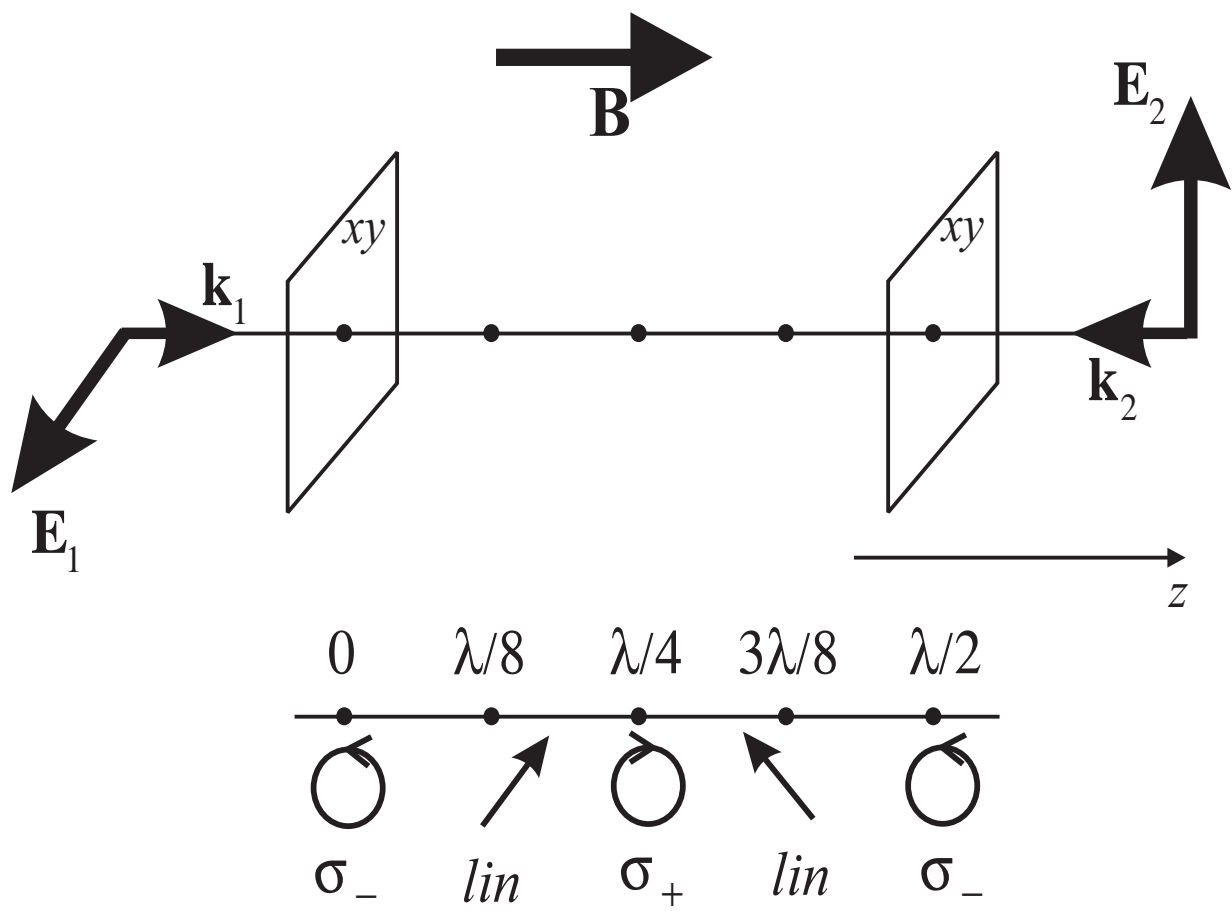


Fig.2.

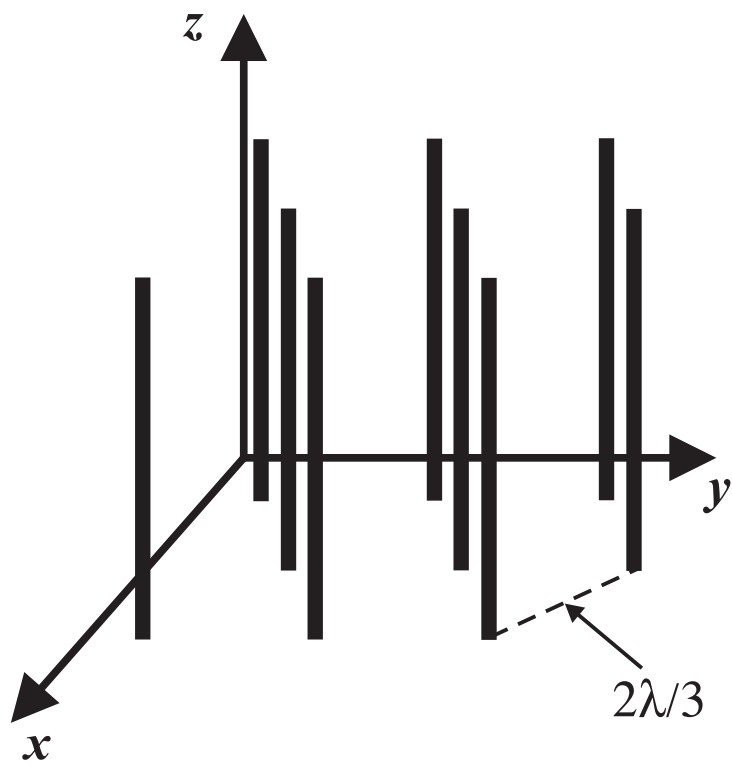
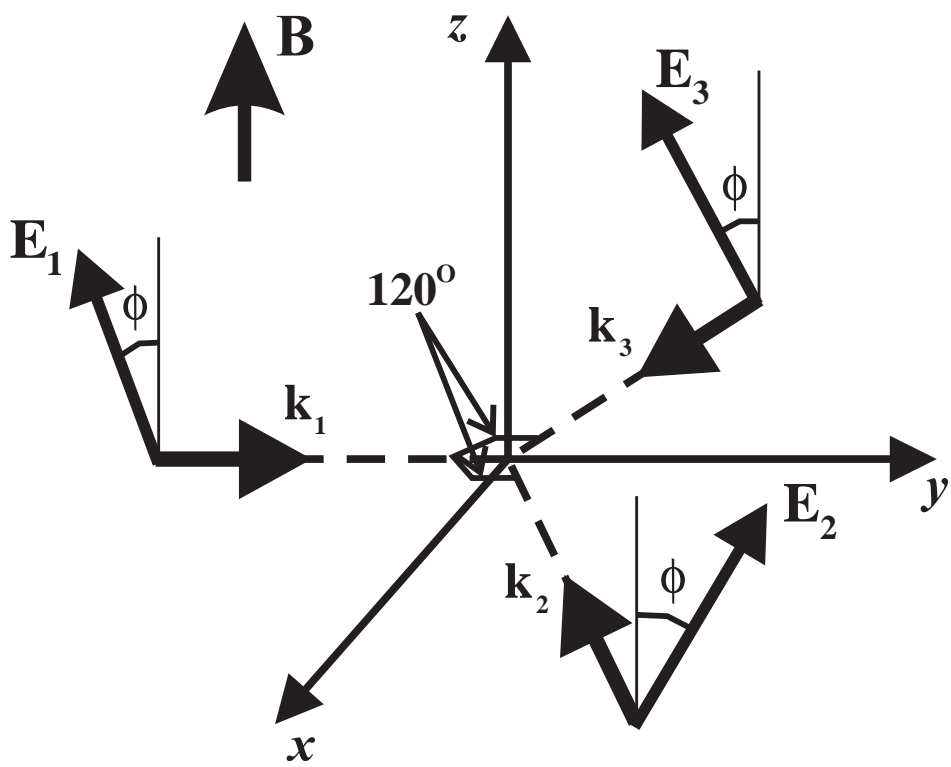


Fig.3.